Minimizing the Number of Transmissions in a Multi-Hop Network for the Dynamical System Filtering Problem and the impact on the mean square error

Jonathan M. Palma O, Leonardo D. P. Carvalho, André M. de Oliveira, Alim P. de C. Gonçalves, Cristian Duran-Faundez.⋆

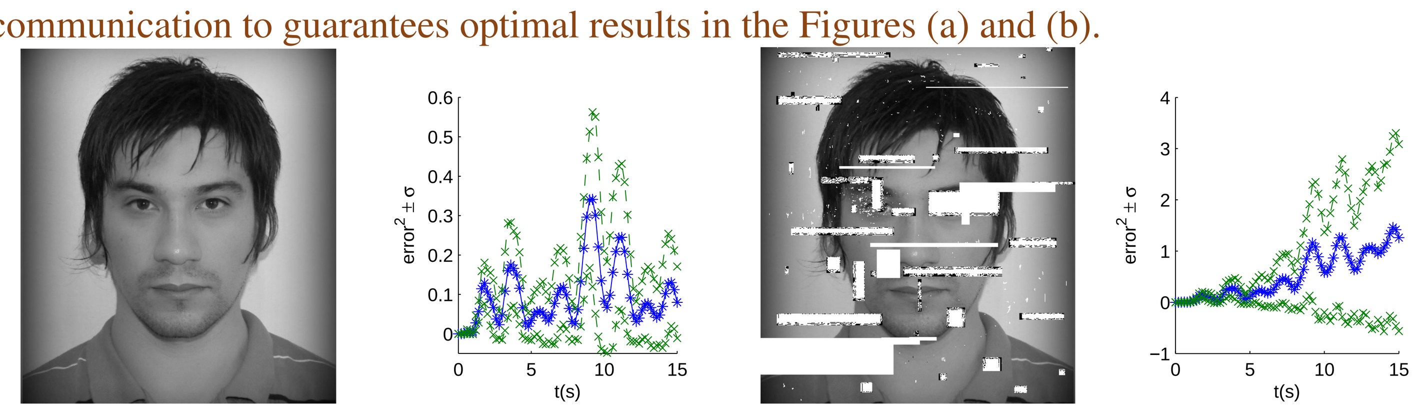
UNICAMP Universidade Estadual de Campinas
UBB Universidad del Bío-Bío⋆

Abstract

In this paper, we study the impact on a dynamical system and the network when using semi reliable communication in a control scheme in a Multi-hop network with the Hop by Hop transport scheme. Specifically, we propose a trade-off between the global number of transmissions and the $\mathcal{H}_\infty$ norm by limiting the maximum number of retransmission per package. Further, we expose the relationship between the $\mathcal{H}_\infty$ norm degradation with the decrease on the expected number of transmissions and the impact on the quadratic error and standard deviation of the estimated state, using a Markov Filter. A practical example is studied, where we show that the relation between the $\mathcal{H}_\infty$ norm, the number of transmissions and the packet loss rate are a function of the maximum allowed number of retransmissions. The results show that there are cases where the overall decreasing in packet retransmission is greater than the increasing in the $\mathcal{H}_\infty$ norm, and thus one may chose the convenient trade-offs between these parameters.[1]

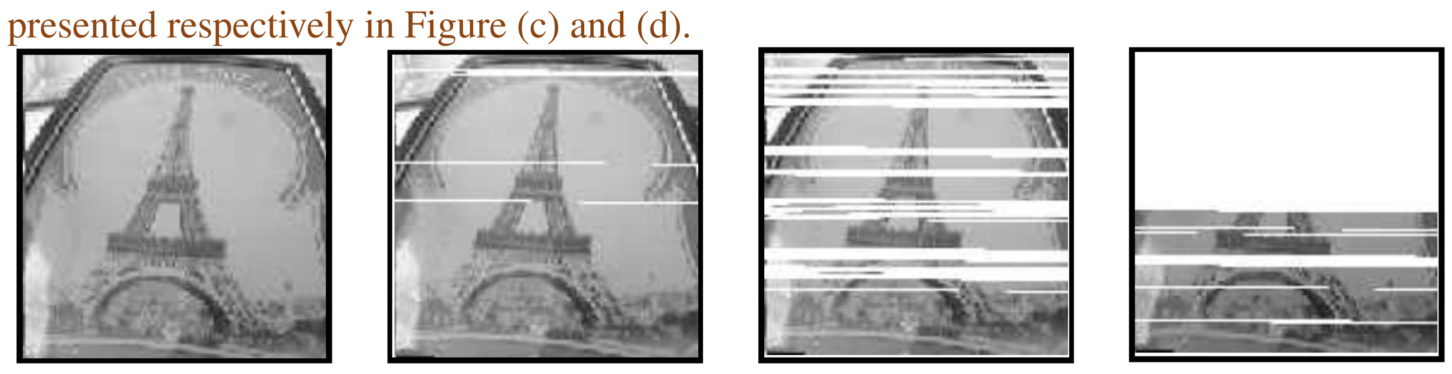
Introduction

The classical control systems are designed on the assumption that every data communication between the interconnected elements, such as sensors, actuators and controllers, has no disturbances in reliability and delivery time. Image transmission in WSN and Control System requires a full-reliable communication to guarantees optimal results in the Figures (a) and (b).

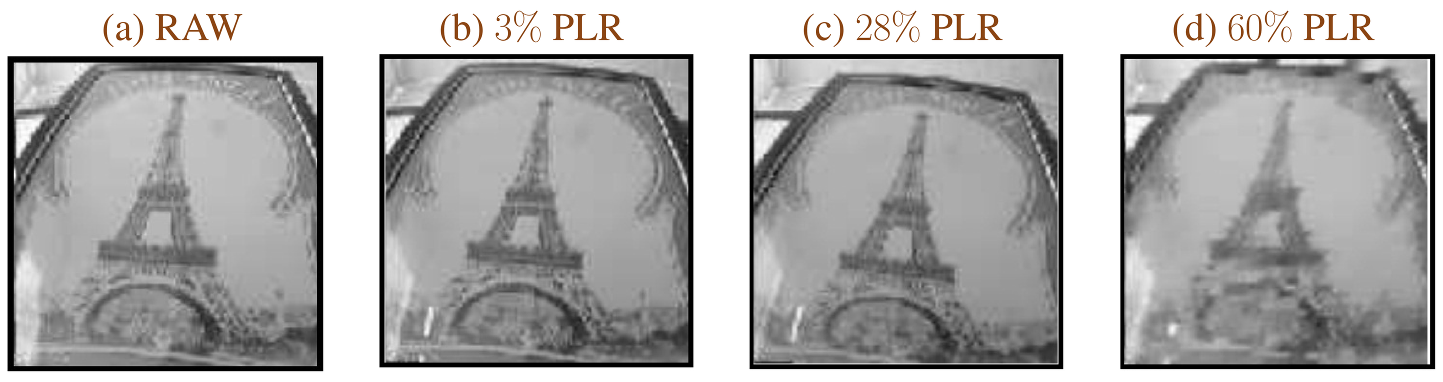


(a) JPG without PLR (b) $error^2$ without PLR (b) JPG with PLR (b) $error^2$ with PLR

Semi-reliable communication **DOES NOT guarantees optimal results** that can lead to those possible scenarios: loss of useful image information, system instability and mean square error diverges, presented respectively in Figure (c) and (d).



(a) RAW (b) 3% PLR (c) 28% PLR (d) 60% PLR

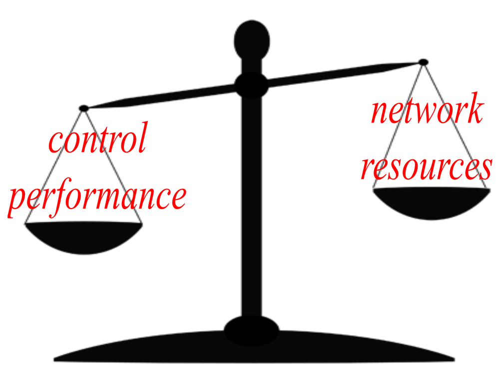


(e) RAW (f) 3% PLR (g) 28% PLR (h) 60% PLR

The image transmission is a popular procedure and because of that fact, there is plenty literature about that specific subject, the minimization of network transmission. A way to achieve this minimization through the semi-reliable network's utilization, where is possible to utilize techniques for that reconstruct the packets lost during the transmission, in consequence the criteria like congestion, collision will suffer a considerable decrease[2].

Trade-OFF in Dynamic System

In this work we propose a similar approach explained about the problem with image transmission. This work is inspired by the described trade-off between retransmissions and data quality in the context of control systems and measurement data transmitted through wireless sensor networks. We propose that the estimation error performance, represented by the $\mathcal{H}_\infty$ norm, can be slightly degraded in order to reduce the number of retransmissions. Dynamic system performance decreases by increasing network resources.



Trade Off $H_\infty/G(TX)$

Performance in dynamic System

The $\mathcal{H}_\infty$ norm represents effectively the maximum cost threshold for the worst disturbance influence on the output $z(k)$. In other words, one ensures that, by computing $\mathcal{H}_\infty$ filters, the $\mathcal{L}_2$ signals $z(k)$ and $w(k)$ relation shown in (1) shall not exceed the threshold, even if the system is excited with the worst disturbance $w \in \mathcal{L}_2$.

The $\mathcal{H}_\infty$ norm of a Markov Jump Linear System is defined as follows:

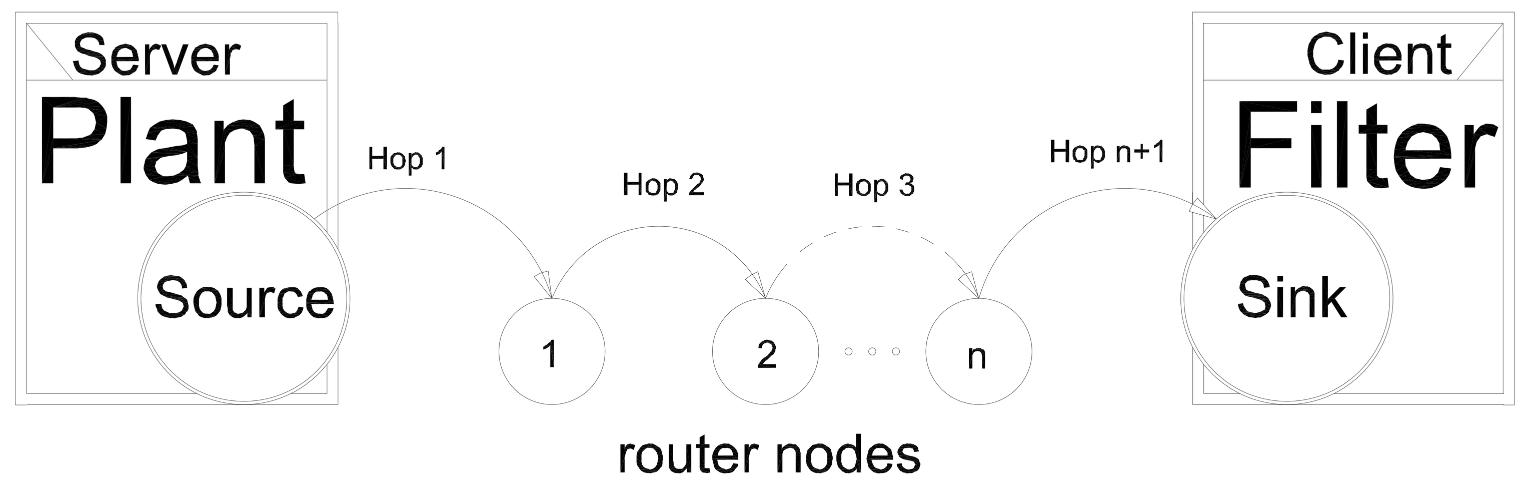
$$||\mathcal{G}||^2_\infty := \sup_{0 \neq w \in \mathcal{L}_2, \theta_0 \in \mathbb{K}} \frac{||z||^2_2}{||w||^2_2}. \quad (1)$$

Cost Interest in the Network

Network parameter performance studied in this work is the expectation of the global number of transmissions. The expectation of the global number of transmission is a measurement proposed that represents the average quantity of transmissions necessary to a single packet of information to reach its destination. Expected results, minimize: **Energy., -Delay. -Congestion.**

Implementing filters in a Multi-Hop network

Figure below shows the filtering problem when the communication is made by a Multi-Hop network.



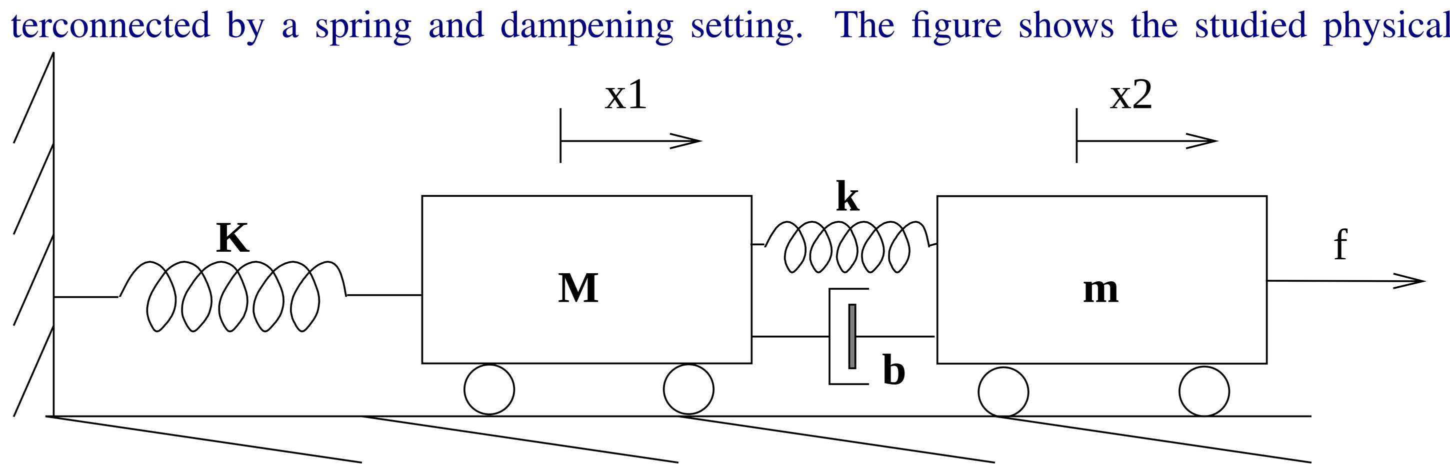
This work uses the Hop-by-Hop model with mechanism transport control through the Source-Sink path, where acknowledgements are transmitted between neighboring nodes in the chain. The analytical expression for calculating the probability of success (P_S) and average number of transmissions for Hop-by-Hop transport model is,

$$P_S = (1 - (1-p)^L)^N \quad (2)$$

Where N is the number of hops, L is the maximum number of transmissions allowed, p is the link data delivery probability, P_F is the probability of failure ($P_F = 1 - P_S$)

Numerical Example

The model used in this example consists of a mass-spring-damp system composed by two cars interconnected by a spring and dampening setting. The figure shows the studied physical system.

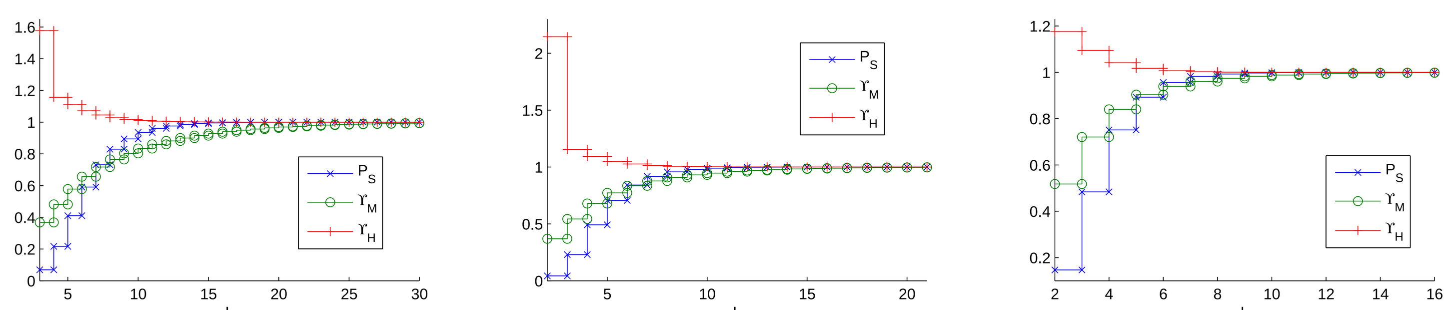


The implemented filter design is an observer filter whose matrices C_{zi} are identity matrices and E_{zi} are null. The x_2 position measurement is transmitted through the discrete Multi-Hop network described.

Results

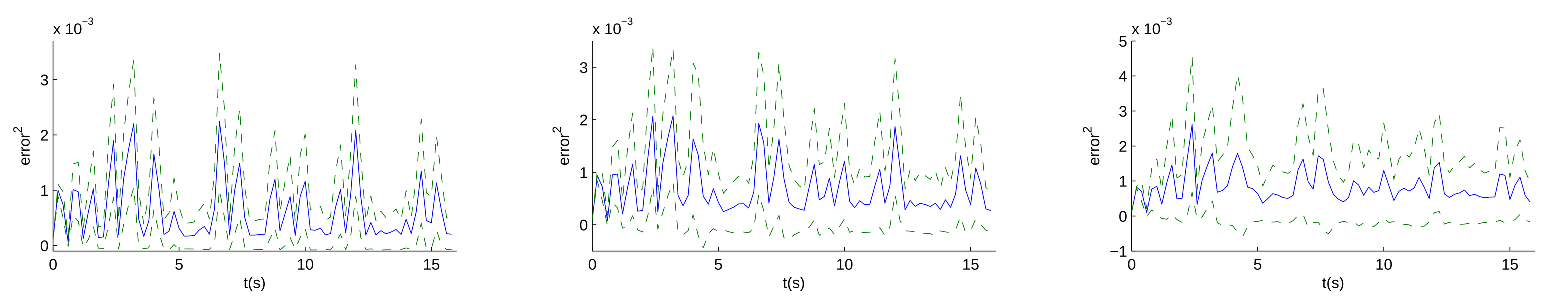
The P_S values depend on the value of p and the number of retransmissions per packet L The mean value of the number of global transmissions on the network is $E(M)$. This value is obtained for each combination of p, L using a Monte Carlo simulation. We define as metric: Υ_H: the rate between the norm obtained with packet loss and the one obtained with the classic filtering without loss and $\Upsilon_M = \frac{\mathcal{E}[M|L<\infty]}{\mathcal{E}[M|L\to\infty]}$: the ratio between the average number of transmission considering a given limit and the average number of transmission without any limit.

The dynamical system's variation measured through the network is exposed in three forms: the first one is the graphic that shows Υ_M, Υ_H and P_S according to the variation of L, for four values of p.



(a) $p = 0.4$ (b) $p = 0.5$ (c) $p = 0.6$

Impact on the Mean Square Error



(a) $L = \infty$ (b) $L = 6$ (c) $L = 4$

The figures above shows the variation metrics in function of L for p. Moreover, Υ_H converges faster than the classical values (without loss of information, Full-reliable communication.) of Υ_M and P_S, which means that for greater values of L, the increment percentage of the $\mathcal{H}_\infty$ norm of the estimated error in the filter is lower than the decrease percentage of the global number of transmissions.

Conclusion

The use of MJLS allows the design of the optimal estimation including the packet loss. For stable systems where the estimated error dynamic is guaranteed stable, the chosen criteria L depends on how much $\mathcal{H}_\infty$ norm the designer is willing to increase in order to minimize the number of transmissions, taking into account also in the estimated error as well.

[1]This poster was funded by FAPESP

[2]This set of images were extracted from Romero and Hernández, 2014